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On Some Derivatives of the Aleph $(\aleph)$-Function

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ABSTRACT: *In the present paper, the authors have established some differential formulae for the Aleph -function. Some special cases of our main results are also given.*
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I. INTRODUCTION

The $\aleph$ - function introduced by Suland et.al. [4] defined and represented in the following form:

$$\aleph[z]=\aleph^{m,n}_{p_i,q_i;\tau_i;r}[z]=\aleph^{m,n}_{p_i,qi;\tau_i;r}\left[z\,\Bigg|\,\begin{matrix}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right]$$

$$=\frac{1}{2\pi w}\int_L \theta(s)z^s ds \tag{1.1}$$

Where $w=\sqrt{-1}$;

$$\theta(s)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{r}\tau_i\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}s\right\}} \tag{1.2}$$

We shall use the following notation:

$$A^*=\left(a_j,\alpha_j\right)_{1,n},[\tau_i\left(a_{ji},\alpha_{ji}\right)]_{n+1,p_i},\ B^*=\left(b_j,\beta_j\right)_{1,m},[\tau_i\left(b_{ji},\beta_{ji}\right)]_{m+1,q_i}$$

II. NOTATIONS AND RESULTS USED

In this paper $\frac{d}{dx}$ is denoted by D_x. Thus

$$D_x^r f(x)=\frac{d^r}{dx^r}f(x) \tag{2.1}$$

$$\left(xD_x\right)^r f(x)=\left(x\frac{d}{dx}\right)^r f(x) \tag{2.2}$$

$$\left(D_x x\right)^r f(x)=\left(\frac{d}{dx}x\right)^r f(x) \tag{2.3}$$

Where the operator xD_x applied on *f (x)* means the function of x is differentiated with respect to x and then multiplied by *x* ; $(xD_x)^r$ means that the operation by xD_x is repeated r times; $D_x x$ applied on *f (x)* means that the function of x is first multiplied by x and then the product is differentiated with respect to x ; D_x^r means that the operation by D_x is repeated r times.

III. MAIN RESULTS

The following results on derivatives of Aleph $(\aleph)$ -function are derived in this section :

Formula 1

$$D_x^r\left\{x^{\lambda}\aleph\left[zx^h\right]\right\} = x^{\lambda-r}\aleph^{m,n+1}_{p_i+1,q_i+1:\tau_i:r}\left[zx^h\left|\begin{matrix}(-\lambda,h),A^*\\ B^*,(r-\lambda,h)\end{matrix}\right.\right] \tag{3.1}$$

For $h>0$ and. $\lambda \in \mathbb{C}$

Formula 2

$$\left(xD_x-k_1\right)...\left(xD_x-k_r\right)\left\{x^{\lambda}\aleph\left[zx^h\right]\right\} = x^{\lambda}\aleph^{m,n+r}_{p_i+r,q_i+r:\tau_i:r}\left[zx^h\left|\begin{matrix}\left(k_j-\lambda,h\right)_{1,r},A^*\\ B^*,\left(1+k_j-\lambda,h\right)_{1,r}\end{matrix}\right.\right] \tag{3.2}$$

Where $\lambda, k_j \in \mathrm{C}(j=1,2,...,k)$ and h is real and positive.

Formula 3

$$\left(D_x x-k_1\right)...\left(D_x x-k_r\right)\left\{x^{\lambda}\aleph\left[zx^h\right]\right\} = x^{\lambda}\aleph^{m,n+r}_{p_i+r,q_i+r:\tau_i:r}\left[zx^h\left|\begin{matrix}\left(k_j-\lambda-1,h\right)_{1,r},A^*\\ B^*,\left(k_j-\lambda,h\right)_{1,r}\end{matrix}\right.\right] \tag{3.3}$$

Where $\lambda, k_j \in \mathrm{C}\,(j=1,2,...,k)$ and h is real and positive.

Proofs:

On using the contour integral (1.1), in the L.H.S. of (3.1), we get

$$\text{L.H.S.}=\left(\frac{d}{dx}\right)^r\left\{x^{\lambda}\,\frac{1}{2\pi w}\int_L \theta(s)\left(zx^h\right)^s ds\right\}$$

Where $\theta(s)$ are given by (1.2).

On applying differentiation under the integral sign, we obtain

$$\text{L.H.S.}=\frac{1}{2\pi w}\int_L \theta(s)\left(zx^h\right)^s ds \quad \times\left\{\left\{\prod_{j=0}^{r-1}\left(\lambda+hs-j\right)\right\}x^{\lambda+hs-r}\right\}ds \tag{3.4}$$

It can easily be shown

$$\prod_{j=0}^{r-1}\left(\lambda+hs-j\right)=\frac{\Gamma\left(1+\lambda+hs\right)}{\Gamma\left(1+\lambda-r+hs\right)} \tag{3.5}$$

On substituting (3.5) in (3.4) and using (1.1), we get the result (3.1)

To prove (3.2), express the left-hand side using the contour integral (1.1), we obtain

$$\text{L.H.S.}=\prod_{j=1}^{r}\left(xD_x-k_j\right)\left\{x^{\lambda}\,\frac{1}{2\pi w}\int_L \theta(s)\left(zx^h\right)^s ds\right\}$$

On applying differentiation under the integral sign, we obtain

$$\text{L.H.S.} \quad =\frac{1}{2\pi w}\int_L \theta(s)z^s\left\{\left\{\prod_{j=1}^{r}\left(\lambda+hs-k_j\right)\right\}x^{\lambda+hs}\right\}ds$$

Taking $\prod_{j=1}^{r}\left(\lambda+hs-k_j\right)=\prod_{j=1}^{r}\frac{\Gamma\left(1+\lambda-k_j+hs\right)}{\Gamma\left(\lambda-k_j+hs\right)}$ and using (1.1), we get the result (3.2)

Similarly, we can obtain the proof of result (3.3)

IV. SPECIAL CASES

(i) When $k_1 = k_2 = ... = k_r = 0$ in (3.2), we get

$$(xD_x)^r \left\{ x^\lambda \aleph\left[zx^h\right]\right\} = x^\lambda \aleph^{m,n+r}_{p_i+r,q_i+r:\tau_i:r}\left[zx^h \left|\begin{matrix}(\lambda,h)_{1,r},A^* \\ B^*,(1-\lambda,h)_{1,r}\end{matrix}\right.\right] \quad (4.1)$$

For $h > 0$ and $\lambda \in \mathrm{C}$

(ii) When $k_1 = k_2 = ... = k_r = 0$ in (3.3), we get

$$(D_x x)^r \left\{ x^\lambda \aleph\left[zx^h\right]\right\} = x^\lambda \aleph^{m,n+r}_{p_i+r,q_i+r:\tau_i:r}\left[zx_h \left|\begin{matrix}(-\lambda-1,h)_{1,r},A^* \\ B^*,(-\lambda,h)_{1,r}\end{matrix}\right.\right] \quad (4.2)$$

For $h > 0$ and $\lambda \in \mathrm{C}$.

(iii) If we set $\tau_i = 1$ in (3.1), the $\aleph$ -function reduces to I -function and we get

$$D_x^r \left\{ x^\lambda I\left[zx^h\right]\right\} = x^{\lambda-r} I^{m,n+1}_{p_i+1,q_i+1:r}\left[zx^h \left|\begin{matrix}(-\lambda,h),A \\ B,(r-\lambda,h)\end{matrix}\right.\right] \quad (4.3)$$

Where $h > 0$, $A = (a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}$, $B = (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}$

From I – Function we can easily obtain various results given in [1, 2, 3].

REFERENCES

[1]. Erdelyi, A. et. al.; Higher Transcendental Functions, vol.I, McGraw-Hill Book Co., New York (1953).

[2]. Fox, C. ;The G and H -function as Fourier kernels. Trans. Amer. Math. Soc.98(1961)395-429.

[3]. Gupta ,K.C. and Jain, U.C.; On the derivative of the H - function, Proc. Indian Acad. Sci. India Sect.38, (1968), 189-192.

[4]. Sudland, N., Baumann, B. and Nonnenmacher, T.F.; Open problem: who knows about the Aleph($\aleph$)-functions? Frac. Calc. Appl. Annl. 1(4),(1998), 401-402.

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